NORTHWESTERN UNIVERSITY Tuesday, May 24, 2005

FIRST TRAINING/SELECTION TEST

NAME: ____________________

WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

Problem A1. Determine the sum of the coefficients of the polynomial that is obtained if we expand the expression $(1 + x - 3x^2)^{2005}$ and reduce like terms.

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Problem A2. The Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, \ldots$ is defined as a sequence whose two first terms are $F_0 = 0$, $F_1 = 1$, and each subsequent term is the sum of the two previous ones: $F_n = F_{n-1} + F_{n-2}$ (for $n \geq 2$). Prove that F_{10^k} is odd for every $k \geq 0$.

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Problem A3. Find the product $\displaystyle\prod_{n=2}^{2005}\left(1-\frac{1}{n^2}\right)$.

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Problem A4. Find the minimum value of $f(x, y, z) = \sqrt{x + y + z}$ subject to the constrains $x, y, z > 0$, $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

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Problem A5. Find the value of the following infinite tower of exponents:

$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}}$$

interpreted as the limit of the infinite sequence 1, $\sqrt{2}$, $\sqrt{2}^{\sqrt{2}}$, $\sqrt{2}^{\sqrt{2}^{\sqrt{2}}}$, ..., NAME: ________________________

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Problem A6. $\overline{RS}$ is the diameter of a semicircle. Two smaller semicircles, $\widehat{RT}$ and $\widehat{TS}$, are drawn on $\overline{RS}$, and their common internal tangent $\overline{AT}$ intersects the large semicircle at A, as shown in the figure. Find the ratio of the area of a semicircle with radius $\overline{AT}$ to the area of the shaded region.