NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A1. Prove that $(\sqrt{5}+2)^{1/3} - (\sqrt{5}-2)^{1/3} = 1$.

NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A2. Let x be a real number. Prove that the sequence a_n with

$$a_n = \sum_{k=1}^{n} \cos(kx)$$

is bounded if and only if x is not a multiple of 2π.

NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A3. For certain $n \times n$-matrices A and B, it is know that $AB = A + B$. Prove that $AB = BA$.

NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A4. Determine whether the following statement is true or false. For every finite set V of positive integers there exists a polynomial P with integer coefficients such that $P(1/n) = n$ for all n in V.

NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A5. Suppose that $a_n > 0$, and $\sum_{n=1}^{\infty} a_n$ converges. Show that there is a sequence $\{b_n\}$ such that $0 < b_n \to \infty$, and $\sum_{n=1}^{\infty} a_n b_n$ converges.

NAME: _____

FALL 2012 NU PUTNAM SELECTION TEST

Problem A6. Let a, b, c the side lengths of a triangle T. Prove that there is a triangle with side lengths a^2, b^2, and c^2 if and only if T is acute (all its angles are acute).