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Problem A1. Find all integer solutions to the system of equations

$$\begin{cases} x^2 - y^2 = 16 \\ x^3 - y^3 = 98 \end{cases}$$

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Problem A2. Prove that 48 divides $n^4 - 1$ if n is not a multiple of 2 or 3.

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Problem A3. We define a sequence $\{a_n\}_{n=1,2,3,\ldots}$ recursively in the following way: $a_1 = 1$, $a_{n+1} = 2(a_n + 1)$ for $n = 1, 2, 3, \ldots$. Find a close form for a_n.

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Problem A4. Find a function $f : \mathbb{R} \to \mathbb{R}$ such that $f \not\equiv 0$ and $f(4x) = f(2x) + f(x)$ for every real x.

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Problem A5. Let A, B, C, D be the following matrices:

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$

Is it possible to obtain the following matrix:

$$E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

by multiplying the given matrices A, B, C, D (in any order and any number of times)?

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Problem A6. Let x, y, z be three real numbers such that $0 < x, y < \pi$, $z = (x + y)/2$. Prove:

$$\sqrt{\frac{\sin x}{x}\frac{\sin y}{y}} \leq \frac{\sin z}{z}\,.$$