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# FALL 2015 NU PUTNAM SELECTION TEST

**Problem A1.** Show that $\log(1+x) > x/(1+x)$ for all $x > 0$.

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**Problem A2.** Define the sequence $a_0 = 0$, $a_{n+1} = \sqrt{\frac{1+a_n}{2}}$ for $n \geq 0$. Find

$$S = \sum_{n=0}^{\infty} \arccos a_n \, .$$

(Note: $y = \arccos x \Leftrightarrow y \in [0, \pi]$ and $\cos y = x$.)

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**Problem A3.** Let $r$ be a real number in the interval $[0, 1)$. Find the sum

$$S = \sum_{k=1}^{\infty} \frac{(-1)^{\lfloor 2^k r \rfloor}}{2^k},$$

where $\lfloor x \rfloor$ = integer part of $x$ = greatest integer less that or equal to $x$.

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**Problem A4.** One hundred passengers board a plane with exactly 100 seats. The first passenger takes a seat at random. The second passenger takes his own seat if it is available, otherwise he takes at random a seat among the available ones. The third passenger takes his own seat if it is available, otherwise he takes at random a seat among the available ones. This process continues until all the 100 passengers have boarded the plane. What is the probability that the last passenger takes his own seat?

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**Problem A5.** Prove that the following divisibility criteria by 61 actually works. Divisibility by 61: Let $n$ be a positive integer. Let $d$ be the rightmost digit of $n$ (in decimal notation), and let $n'$ be the number obtained by removing from $n$ its rightmost digit (if $n$ has only one digit then $n' = 0$). Replace $n$ with $n' - 6d$. Repeat those steps while the result is still a positive integer. If the process ends in zero then the original number is divisible by 61, otherwise it is not. Example for $n = 21045$: $2104 - 6 \cdot 5 = 2074$, $207 - 6 \cdot 4 = 183$, $18 - 6 \cdot 3 = 0$. Hence 21045 is divisible by 61.

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**Problem A6.** Flip a fair coin until heads turns out twice consecutively. What is the expected number of flips?