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Problem A1. Assume that a rectangle of dimensions a and b contains inside it another rectangle of dimensions a' and b'. Prove that $a' + b' < a + b$.

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Problem A2. We place $4n$ points uniformly on a circle. Then we paint any $2n$ of them in red and the other $2n$ points in blue. Prove that regardless of which points we have painted with each color, there is always a straight line that divides the circle in half leaving exactly n red points and n blue points at each side.

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Problem A3. Show that for every positive integer n, $4^n + 6n - 1$ is a multiple of 9.

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Problem A4. Prove $\displaystyle\int_0^{\frac{\pi}{2}} e^{\sin x}\,dx \geq \frac{\pi}{2}(e-1)$.

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Problem A5. A number n has 250 positive divisors, sorted and indexed in increasing order: $1 = d_1 < d_2 < d_3 < \cdots < d_{250} = n$. Ted is allowed to pick two indices i and j ($1 \leq i, j \leq 249$), with the condition that $i + j \neq 251$, and he is given in return divisors d_i and d_j. Show that Ted can always find the value of n by picking appropriately those two indices.